Sears and Zemansky's University Physics with Modern Physics Technology Update
Hugh D. Young   Roger A. Freedman
Thirteenth Edition

PEARSON

SEARS & ZEMANSKY'S
UNIVERSITY PHYSICS
WITH MODERN PHYSICS

TECHNOLOGY UPDATE

YOUNG AND FREEDMAN
13TH EDITION

Pearson New International Edition

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HOW TO SUCCEED IN PHYSICS BY REALLY TRYING

Mark Hollabaugh *Normandale Community College*

Physics encompasses the large and the small, the old and the new. From the atom to galaxies, from electrical circuitry to aerodynamics, physics is very much a part of the world around us. You probably are taking this introductory course in calculus-based physics because it is required for subsequent courses you plan to take in preparation for a career in science or engineering. Your professor wants you to learn physics and to enjoy the experience. He or she is very interested in helping you learn this fascinating subject. That is part of the reason your professor chose this textbook for your course. That is also the reason Drs. Young and Freedman asked me to write this introductory section. We want you to succeed!

The purpose of this section of *University Physics* is to give you some ideas that will assist your learning. Specific suggestions on how to use the textbook will follow a brief discussion of general study habits and strategies.

Preparation for This Course

If you had high school physics, you will probably learn concepts faster than those who have not because you will be familiar with the language of physics. If English is a second language for you, keep a glossary of new terms that you encounter and make sure you understand how they are used in physics. Likewise, if you are farther along in your mathematics courses, you will pick up the mathematical aspects of physics faster. Even if your mathematics is adequate, you may find a book such as Arnold D. Pickar's *Preparing for General Physics: Math Skill Drills and Other Useful Help (Calculus Version)* to be useful. Your professor may actually assign sections of this math review to assist your learning.

Learning to Learn

Each of us has a different learning style and a preferred means of learning. Understanding your own learning style will help you to focus on aspects of physics that may give you difficulty and to use those components of your course that will help you overcome the difficulty. Obviously you will want to spend more time on those aspects that give you the most trouble. If you learn by hearing, lectures will be very important. If you learn by explaining, then working with other students will be useful to you. If solving problems is difficult for you, spend more time learning how to solve problems. Also, it is important to understand and develop good study habits. Perhaps the most important thing you can do for yourself is to set aside adequate, regularly scheduled study time in a distraction-free environment.

Answer the following questions for yourself:
- Am I able to use fundamental mathematical concepts from algebra, geometry and trigonometry? (If not, plan a program of review with help from your professor.)
- In similar courses, what activity has given me the most trouble? (Spend more time on this.) What has been the easiest for me? (Do this first; it will help to build your confidence.)

- Do I understand the material better if I read the book before or after the lecture? (You may learn best by skimming the material, going to lecture, and then undertaking an in-depth reading.)
- Do I spend adequate time in studying physics? (A rule of thumb for a class like this is to devote, on the average, 2.5 hours out of class for each hour in class. For a course meeting 5 hours each week, that means you should spend about 10 to 15 hours per week studying physics.)
- Do I study physics every day? (Spread that 10 to 15 hours out over an entire week!) At what time of the day am I at my best for studying physics? (Pick a specific time of the day and stick to it.)
- Do I work in a quiet place where I can maintain my focus? (Distractions will break your routine and cause you to miss important points.)

Working with Others

Scientists or engineers seldom work in isolation from one another but rather work cooperatively. You will learn more physics and have more fun doing it if you work with other students. Some professors may formalize the use of cooperative learning or facilitate the formation of study groups. You may wish to form your own informal study group with members of your class who live in your neighborhood or dorm. If you have access to e-mail, use it to keep in touch with one another. Your study group is an excellent resource when reviewing for exams.

Lectures and Taking Notes

An important component of any college course is the lecture. In physics this is especially important because your professor will frequently do demonstrations of physical principles, run computer simulations, or show video clips. All of these are learning activities that will help you to understand the basic principles of physics. Don't miss lectures, and if for some reason you do, ask a friend or member of your study group to provide you with notes and let you know what happened.

Take your class notes in outline form, and fill in the details later. It can be very difficult to take word for word notes, so just write down key ideas. Your professor may use a diagram from the textbook. Leave a space in your notes and just add the diagram later. After class, edit your notes, filling in any gaps or omissions and noting things you need to study further. Make references to the textbook by page, equation number, or section number.

Make sure you ask questions in class, or see your professor during office hours. Remember the only "dumb" question is the one that is not asked. Your college may also have teaching assistants or peer tutors who are available to help you with difficulties you may have.

Examinations

Taking an examination is stressful. But if you feel adequately prepared and are well-rested, your stress will be lessened. Preparing for an exam is a continual process; it begins the moment the last exam is over. You should immediately go over the exam and understand any mistakes you made. If you worked a problem and made substantial errors, try this: Take a piece of paper and divide it down the middle with a line from top to bottom. In one column, write the proper solution to the problem. In the other column, write what you did and why, if you know, and why your solution was incorrect. If you are uncertain why you made your mistake, or how to avoid making it again, talk with your professor. Physics continually builds on fundamental ideas and it is important to correct any misunderstandings immediately. *Warning:* While cramming at the last minute may get you through the present exam, you will not adequately retain the concepts for use on the next exam.

PREFACE

This book is the product of more than six decades of leadership and innovation in physics education. When the first edition of *University Physics* by Francis W. Sears and Mark W. Zemansky was published in 1949, it was revolutionary among calculus-based physics textbooks in its emphasis on the fundamental principles of physics and how to apply them. The success of *University Physics* with generations of several million students and educators around the world is a testament to the merits of this approach, and to the many innovations it has introduced subsequently.

In preparing this new Thirteenth Edition, we have further enhanced and developed *University Physics* to assimilate the best ideas from education research with enhanced problem-solving instruction, pioneering visual and conceptual pedagogy, the first systematically enhanced problems, and the most pedagogically proven and widely used online homework and tutorial system in the world.

New to This Edition

- Included in each chapter, **Bridging Problems** provide a transition between the single-concept Examples and the more challenging end-of-chapter problems. Each Bridging Problem poses a difficult, multiconcept problem, which often incorporates physics from earlier chapters. In place of a full solution, it provides a skeleton **Solution Guide** consisting of questions and hints, which helps train students to approach and solve challenging problems with confidence.

  Video Tutor Demo

- **All Examples, Conceptual Examples, and Problem-Solving Strategies are revised** to enhance conciseness and clarity for today's students.
- The **core modern physics chapters** (Chapters 38–41) are revised extensively to provide a more idea-centered, less historical approach to the material. Chapters 42–44 are also revised significantly.
- **The fluid mechanics chapter now precedes the chapters on gravitation and periodic motion,** so that the latter immediately precedes the chapter on mechanical waves.
- **Additional bioscience applications** appear throughout the text, mostly in the form of marginal photos with explanatory captions, to help students see how physics is connected to many breakthroughs and discoveries in the biosciences.
- The **text has been streamlined** for tighter and more focused language.
- **Using data from MasteringPhysics, changes to the end-of-chapter content** include the following:
  - **15%–20% of problems are new.**
  - The number and level of **calculus-requiring problems** has been increased.
  - Most chapters include **five to seven biosciences-related problems.**
  - The number of **cumulative problems** (those incorporating physics from earlier chapters) has been increased.
- **Over 70 PhET simulations** are linked to the Pearson eText and provided in the Study Area of the MasteringPhysics website (with icons in the print text). These powerful simulations allow students to interact productively with the physics concepts they are learning. PhET clicker questions are also included on the Instructor Resource DVD.
- **Video Tutors bring key content to life throughout the text:**
  - **Dozens of Video Tutors feature "pause-and-predict" demonstrations of key physics concepts** and incorporate assessment as the student progresses to actively engage the student in understanding the key conceptual ideas underlying the physics principles.

Standard, Extended, and Three-Volume Editions

With MasteringPhysics:
- **Standard Edition:** Chapters 1–37 (ISBN 978-0-321-69688-5)
- **Extended Edition:** Chapters 1–44 (ISBN 978-0-321-67546-0)

Without MasteringPhysics:
- **Standard Edition:** Chapters 1–37 (ISBN 978-0-321-69689-2)
- **Extended Edition:** Chapters 1–44 (ISBN 978-0-321-69686-1)
- **Volume 1:** Chapters 1–20 (ISBN 978-0-321-73338-2)
- **Volume 2:** Chapters 21–37 (ISBN 978-0-321-75121-8)
- **Volume 3:** Chapters 37–44 (ISBN 978-0-321-75120-1)

- **Every Worked Example in the book is accompanied by a Video Tutor Solution** that walks students through the problem-solving process, providing a virtual teaching assistant on a round-the-clock basis.
- **All of these Video Tutors play directly through links within the Pearson eText.** Many also appear in the Study Area within MasteringPhysics.

Key Features of *University Physics*

- Deep and extensive **problem sets** cover a wide range of difficulty and exercise both physical understanding and problem-solving expertise. Many problems are based on complex real-life situations.
- This text offers a larger number of **Examples** and **Conceptual Examples** than any other leading calculus-based text, allowing it to explore problem-solving challenges not addressed in other texts.
- A research-based **problem-solving approach (Identify, Set Up, Execute, Evaluate)** is used not just in every Example but also in the Problem-Solving Strategies and throughout the Student and Instructor Solutions Manuals and the Study Guide. This consistent approach teaches students to tackle problems thoughtfully rather than cutting straight to the math.
- **Problem-Solving Strategies** coach students in how to approach specific types of problems.
- The **Figures** use a simplified graphical style to focus on the physics of a situation, and they incorporate **explanatory annotation.** Both techniques have been demonstrated to have a strong positive effect on learning.
- Figures that illustrate Example solutions often take the form of black-and-white **pencil sketches,** which directly represent what a student should draw in solving such a problem.
- The popular **Caution paragraphs** focus on typical misconceptions and student problem areas.
- End-of-section **Test Your Understanding** questions let students check their grasp of the material and use a multiple-choice or ranking-task format to probe for common misconceptions.
- **Visual Summaries** at the end of each chapter present the key ideas in words, equations, and thumbnail pictures, helping students to review more effectively.

Instructor Supplements

Note: For convenience, all of the following instructor supplements (except for the Instructor Resource DVD) can be downloaded from the Instructor Area, accessed via the left-hand navigation bar of MasteringPhysics (www.masteringphysics.com).

Instructor Solutions, prepared by A. Lewis Ford (Texas A&M University) and Wayne Anderson, contain complete and detailed solutions to all end-of-chapter problems. All solutions follow consistently the same Identify/Set Up/Execute/Evaluate problem-solving framework used in the textbook. Download only from the MasteringPhysics Instructor Area or from the Instructor Resource Center (www.pearsonhighered.com/irc).

The cross-platform **Instructor Resource DVD** (ISBN 978-0-321-69661-8) provides a comprehensive library of more than 420 applets from ActivPhysics OnLine as well as all line figures from the textbook in JPEG format. In addition, all the key equations, problem-solving strategies, tables, and chapter summaries are provided in editable Word format. In-class weekly multiple-choice questions for use with various Classroom Response Systems (CRS) are also provided, based on the Test Your Understanding questions in the text. Lecture outlines in PowerPoint are also included along with over 70 PhET simulations.

MasteringPhysics® (www.masteringphysics.com) is the most advanced, educationally effective, and widely used physics homework and tutorial system in the world. Eight years in development, it provides instructors with a library of extensively pre-tested end-of-chapter problems and rich, multipart, multistep tutorials that incorporate a wide variety of answer types, wrong answer feedback, individualized help (comprising hints or simpler sub-problems upon request), all driven by the largest metadatabase of student problem-solving in the world. NSF-sponsored published research (and subsequent studies) show that Mastering-Physics has dramatic educational results. MasteringPhysics allows instructors to build wide-ranging homework assignments of just the right difficulty and length and provides them with efficient tools to analyze both class trends, and the work of any student in unprecedented detail.

MasteringPhysics routinely provides instant and individualized feedback and guidance to more than 100,000 students every day. A wide range of tools and support make MasteringPhysics fast and easy for instructors and students to learn to use. Extensive class tests show that by the end of their course, an unprecedented eight of nine students recommend MasteringPhysics as their preferred way to study physics and do homework.

MasteringPhysics enables instructors to:
• Quickly build homework assignments that combine regular end-of-chapter problems and tutoring (through additional multi-step tutorial problems that offer wrong-answer feedback and simpler problems upon request).
• Expand homework to include the widest range of automatically graded activities available—from numerical problems with randomized values, through algebraic answers, to free-hand drawing.
• Choose from a wide range of nationally pre-tested problems that provide accurate estimates of time to complete and difficulty.
• After an assignment is completed, quickly identify not only the problems that were the trickiest for students but the individual problem types where students had trouble.
• Compare class results against the system's worldwide average for each problem assigned, to identify issues to be addressed with just-in-time teaching.
• Check the work of an individual student in detail, including time spent on each problem, what wrong answers they submitted at each step, how much help they asked for, and how many practice problems they worked.

ActivPhysics OnLine™ (which is accessed through the Study Area within www.masteringphysics.com) provides a comprehensive library of more than 420 tried and tested ActivPhysics applets updated for web delivery using the latest online technologies. In addition, it provides a suite of highly regarded applet-based tutorials developed by education pioneers Alan Van Heuvelen and Paul D'Alessandris. Margin icons throughout the text direct students to specific exercises that complement the textbook discussion.

The online exercises are designed to encourage students to confront misconceptions, reason qualitatively about physical processes, experiment quantitatively, and learn to think critically. The highly acclaimed ActivPhysics OnLine companion workbooks help students work through complex concepts and understand them more clearly. More than 420 applets from the ActivPhysics OnLine library are also available on the Instructor Resource DVD for this text.

The **Test Bank** contains more than 2,000 high-quality problems, with a range of multiple-choice, true/false, short-answer, and regular homework-type questions. Test files are provided both in TestGen (an easy-to-use, fully networkable program for creating and editing quizzes and exams) and Word format. Download only from the MasteringPhysics Instructor Area or from the Instructor Resource Center (www.pearsonhighered.com/irc).

Five Easy Lessons: Strategies for Successful Physics Teaching (ISBN 978-0-805-38702-5) by Randall D. Knight (California Polytechnic State University, San Luis Obispo) is packed with creative ideas on how to enhance any physics course. It is an invaluable companion for both novice and veteran physics instructors.

Student Supplements

The **Study Guide** by Laird Kramer reinforces the text's emphasis on problem-solving strategies and student misconceptions. The *Study Guide for Volume 1* (ISBN 978-0-321-69665-6) covers Chapters 1–20, and the *Study Guide for Volumes 2 and 3* (ISBN 978-0-321-69669-4) covers Chapters 21–44.

The **Student Solutions Manual** by Lewis Ford (Texas A&M University) and Wayne Anderson contains detailed, step-by-step solutions to more than half of the odd-numbered end-of-chapter problems from the textbook. All solutions follow consistently the same Identify/Set Up/Execute/Evaluate problem-solving framework used in the textbook. The *Student Solutions Manual for Volume 1* (ISBN 978-0-321-69668-7) covers Chapters 1–20, and the *Student Solutions Manual for Volumes 2 and 3* (ISBN 978-0-321-69667-0) covers Chapters 21–44.

MasteringPhysics® (www.masteringphysics.com) is a homework, tutorial, and assessment system based on years of research into how students work physics problems and precisely where they need help. Studies show that students who use MasteringPhysics significantly increase their scores compared to hand-written homework. MasteringPhysics achieves this improvement by providing students with instantaneous feedback specific to their wrong answers, simpler sub-problems upon request when they get stuck, and partial credit for their method(s). This individualized, 24/7 Socratic tutoring is recommended by nine out of ten students to their peers as the most effective and time-efficient way to study.

Pearson eText is available through MasteringPhysics, either automatically when MasteringPhysics is packaged with new books, or available as a purchased upgrade online. Allowing students access to the text wherever they have access to the Internet, Pearson eText comprises the full text, including figures that can be enlarged for better viewing. With eText, students are also able to pop up definitions and terms to help with vocabulary and the reading of the material. Students can also take notes in eText using the annotation feature at the top of each page.

Pearson Tutor Services (www.pearsontutorservices.com). Each student's subscription to MasteringPhysics also contains complimentary access to Pearson Tutor Services, powered by Smarthinking, Inc. By logging in with their MasteringPhysics ID and password, students will be connected to highly qualified e-instructors who provide additional interactive online tutoring on the major concepts of physics. Some restrictions apply; offer subject to change.

ActivPhysics OnLine™ (which is accessed through the Study Area within www.masteringphysics.com) provides students with a suite of highly regarded applet-based tutorials (see above). The following workbooks help students work through complex concepts and understand them more clearly.

ActivPhysics OnLine Workbook, Volume 1: Mechanics * Thermal Physics * Oscillations & Waves (978-0-805-39060-5)

ActivPhysics OnLine Workbook, Volume 2: Electricity & Magnetism * Optics * Modern Physics (978-0-805-39061-2)

Acknowledgments

We would like to thank the hundreds of reviewers and colleagues who have offered valuable comments and suggestions over the life of this textbook. The continuing success of *University Physics* is due in large measure to their contributions.

Please Tell Us What You Think!

We welcome communications from students and professors, especially concerning errors or deficiencies that you find in this edition. We have devoted a lot of time and effort to writing the best book we know how to write, and we hope it will help you to teach and learn physics. In turn, you can help us by letting us know what still needs to be improved! Please feel free to contact us either electronically or by ordinary mail. Your comments will be greatly appreciated.

December 2010

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UNITS, PHYSICAL QUANTITIES, AND VECTORS

Ralph Wetmore/Getty Images

? Being able to predict the path of a thunderstorm is essential for minimizing the damage it does to lives and property. If a thunderstorm is moving at 20 km/h in a direction 53° north of east, how far north does the thunderstorm move in 1 h?

LEARNING GOALS

By studying this chapter, you will learn:

- Three fundamental quantities of physics and the units physicists use to measure them.

- How to keep track of significant figures in your calculations.

- The difference between scalars and vectors, and how to add and sub- tract vectors graphically.

- What the components of a vector are, and how to use them in calculations.

- What unit vectors are, and how to use them with components to describe vectors.

- Two ways of multiplying vectors.

Physics is one of the most fundamental of the sciences. Scientists of all dis- ciplines use the ideas of physics, including chemists who study the struc- ture of molecules, paleontologists who try to reconstruct how dinosaurs walked, and climatologists who study how human activities affect the atmos- phere and oceans. Physics is also the foundation of all engineering and technol- ogy. No engineer could design a flat-screen TV, an interplanetary spacecraft, or even a better mousetrap without first understanding the basic laws of physics.

The study of physics is also an adventure. You will find it challenging, some- times frustrating, occasionally painful, and often richly rewarding. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. You will come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves.

In this chapter, we'll go over some important preliminaries that we'll need throughout our study. We'll discuss the nature of physical theory and the use of idealized models to represent physical systems. We'll introduce the systems of units used to describe physical quantities and discuss ways to describe the accu- racy of a number. We'll look at examples of problems for which we can't (or don't want to) find a precise answer, but for which rough estimates can be useful and interesting. Finally, we'll study several aspects of vectors and vector algebra. Vectors will be needed throughout our study of physics to describe and analyze physical quantities, such as velocity and force, that have direction as well as magnitude.

9

1 The Nature of Physics

Physics is an *experimental* science. Physicists observe the phenomena of nature and try to find patterns that relate these phenomena. These patterns are called physical theories or, when they are very well established and widely used, physical laws or principles.

CAUTION **The meaning of the word "theory"** Calling an idea a theory does *not* mean that it's just a random thought or an unproven concept. Rather, a theory is an explanation of natural phenomena based on observation and accepted fundamental principles. An example is the well-established theory of biological evolution, which is the result of extensive research and observation by generations of biologists.

To develop a physical theory, a physicist has to learn to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results. Figure 1 shows two famous facilities used for physics experiments.

Legend has it that Galileo Galilei (1564–1642) dropped light and heavy objects from the top of the Leaning Tower of Pisa (Fig. 1a) to find out whether their rates of fall were the same or different. From examining the results of his experiments (which were actually much more sophisticated than in the legend), he made the inductive leap to the principle, or theory, that the acceleration of a falling body is independent of its weight.

The development of physical theories such as Galileo's often takes an indirect path, with blind alleys, wrong guesses, and the discarding of unsuccessful theories in favor of more promising ones. Physics is not simply a collection of facts and principles; it is also the *process* by which we arrive at general principles that describe how the physical universe behaves.

No theory is ever regarded as the final or ultimate truth. The possibility always exists that new observations will require that a theory be revised or discarded. It is in the nature of physical theory that we can disprove a theory by finding behavior that is inconsistent with it, but we can never prove that a theory is always correct.

Getting back to Galileo, suppose we drop a feather and a cannonball. They certainly do *not* fall at the same rate. This does not mean that Galileo was wrong; it means that his theory was incomplete. If we drop the feather and the cannonball *in a vacuum* to eliminate the effects of the air, then they do fall at the same rate. Galileo's theory has a **range of validity:** It applies only to objects for which the force exerted by the air (due to air resistance and buoyancy) is much less than the weight. Objects like feathers or parachutes are clearly outside this range.

Often a new development in physics extends a principle's range of validity. Galileo's analysis of falling bodies was greatly extended half a century later by Newton's laws of motion and law of gravitation.

1 Two research laboratories. **(a)** According to legend, Galileo investigated falling bodies by dropping them from the Leaning Tower in Pisa, Italy, and he studied pendulum motion by observing the swinging of the chandelier in the adjacent cathedral. **(b)** The Large Hadron Collider (LHC) in Geneva, Switzerland, the world's largest particle accelerator, is used to explore the smallest and most fundamental constituents of matter. This photo shows a portion of one of the LHC's detectors (note the worker on the yellow platform).

(a)

Shutterstock

(b)

CERN/European Organization for Nuclear Research

2 Solving Physics Problems

At some point in their studies, almost all physics students find themselves thinking, "I understand the concepts, but I just can't solve the problems." But in physics, truly understanding a concept *means* being able to apply it to a variety of problems. Learning how to solve problems is absolutely essential; you don't *know* physics unless you can *do* physics.

How do you learn to solve physics problems? You will find *Problem-Solving Strategies* that offer techniques for setting up and solving problems efficiently and accurately. Following each *Problem-Solving Strategy* are one or more worked *Examples* that show these techniques in action. (The *Problem-Solving Strategies* will also steer you away from some *incorrect* techniques that you may be tempted to use.) You'll also find additional examples that aren't associated with a particular *Problem-Solving Strategy*. In addition,

in this text you'll find a *Bridging Problem* that uses more than one of the key ideas from the chapter. Study these strategies and problems carefully, and work through each example for yourself on a piece of paper.

Different techniques are useful for solving different kinds of physics problems, which is why this text offers dozens of *Problem-Solving Strategies.* No matter what kind of problem you're dealing with, however, there are certain key steps that you'll always follow. (These same steps are equally useful for problems in math, engineering, chemistry, and many other fields.) In this text we've organized these steps into four stages of solving a problem.

All of the *Problem-Solving Strategies* and *Examples* will follow these four steps. (In some cases we will combine the first two or three steps.) We encourage you to follow these same steps when you solve problems yourself. You may find it useful to remember the acronym **I SEE**—short for *Identify, Set up, Execute,* and *Evaluate.*

Problem-Solving Strategy 1   **Solving Physics Problems**

IDENTIFY *the relevant concepts:* Use the physical conditions stated in the problem to help you decide which physics concepts are relevant. Identify the **target variables** of the problem—that is, the quantities whose values you're trying to find, such as the speed at which a projectile hits the ground, the intensity of a sound made by a siren, or the size of an image made by a lens. Identify the known quantities, as stated or implied in the problem. This step is essential whether the problem asks for an algebraic expression or a numerical answer.

SET UP *the problem:* Given the concepts you have identified and the known and target quantities, choose the equations that you'll use to solve the problem and decide how you'll use them. Make sure that the variables you have identified correlate exactly with those in the equations. If appropriate, draw a sketch of the situation described in the problem. (Graph paper, ruler, protractor, and compass will help you make clear, useful sketches.) As best you can,

estimate what your results will be and, as appropriate, predict what the physical behavior of a system will be. The worked examples in this text include tips on how to make these kinds of estimates and predictions. If this seems challenging, don't worry—you'll get better with practice!

EXECUTE *the solution:* This is where you "do the math." Study the worked examples to see what's involved in this step.

EVALUATE *your answer:* Compare your answer with your estimates, and reconsider things if there's a discrepancy. If your answer includes an algebraic expression, assure yourself that it represents what would happen if the variables in it were taken to extremes. For future reference, make note of any answer that represents a quantity of particular significance. Ask yourself how you might answer a more general or more difficult version of the problem you have just solved.

Idealized Models

In everyday conversation we use the word "model" to mean either a small-scale replica, such as a model railroad, or a person who displays articles of clothing (or the absence thereof). In physics a **model** is a simplified version of a physical system that would be too complicated to analyze in full detail.

For example, suppose we want to analyze the motion of a thrown baseball (Fig. 2a). How complicated is this problem? The ball is not a perfect sphere (it has raised seams), and it spins as it moves through the air. Wind and air resistance influence its motion, the ball's weight varies a little as its distance from the center of the earth changes, and so on. If we try to include all these things, the analysis gets hopelessly complicated. Instead, we invent a simplified version of the problem. We neglect the size and shape of the ball by representing it as a point object, or **particle.** We neglect air resistance by making the ball move in a vacuum, and we make the weight constant. Now we have a problem that is simple enough to deal with (Fig. 2b).

We have to overlook quite a few minor effects to make an idealized model, but we must be careful not to neglect too much. If we ignore the effects of gravity completely, then our model predicts that when we throw the ball up, it will go in a straight line and disappear into space. A useful model is one that simplifies a problem enough to make it manageable, yet keeps its essential features.

2 To simplify the analysis of (a) a baseball in flight, we use (b) an idealized model.

(a) A real baseball in flight

Baseball spins and has a complex shape.

Air resistance and wind exert forces on the ball.

Direction of motion

Gravitational force on ball depends on altitude.

(b) An idealized model of the baseball

Baseball is treated as a point object (particle).

No air resistance.

Direction of motion

Gravitational force on ball is constant.
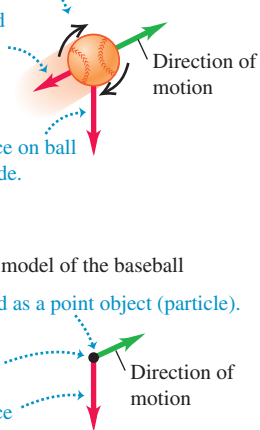
The validity of the predictions we make using a model is limited by the validity of the model. For example, Galileo's prediction about falling bodies (see Section 1) corresponds to an idealized model that does not include the effects of air resistance. This model works fairly well for a dropped cannonball, but not so well for a feather.

Idealized models play a crucial role in this text. Watch for them in discussions of physical theories and their applications to specific problems.

3 Standards and Units

As we learned in Section 1, physics is an experimental science. Experiments require measurements, and we generally use numbers to describe the results of measurements. Any number that is used to describe a physical phenomenon quantitatively is called a **physical quantity.** For example, two physical quantities that describe you are your weight and your height. Some physical quantities are so fundamental that we can define them only by describing how to measure them. Such a definition is called an **operational definition.** Two examples are measuring a distance by using a ruler and measuring a time interval by using a stopwatch. In other cases we define a physical quantity by describing how to calculate it from other quantities that we *can* measure. Thus we might define the average speed of a moving object as the distance traveled (measured with a ruler) divided by the time of travel (measured with a stopwatch).

When we measure a quantity, we always compare it with some reference standard. When we say that a Ferrari 458 Italia is 4.53 meters long, we mean that it is 4.53 times as long as a meter stick, which we define to be 1 meter long. Such a standard defines a **unit** of the quantity. The meter is a unit of distance, and the second is a unit of time. When we use a number to describe a physical quantity, we must always specify the unit that we are using; to describe a distance as simply "4.53" wouldn't mean anything.

To make accurate, reliable measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations. The system of units used by scientists and engineers around the world is commonly called "the metric system," but since 1960 it has been known officially as the **International System,** or **SI** (the abbreviation for its French name, *Système International*).

Time

From 1889 until 1967, the unit of time was defined as a certain fraction of the mean solar day, the average time between successive arrivals of the sun at its highest point in the sky. The present standard, adopted in 1967, is much more precise. It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the cesium atom. When bombarded by microwaves of precisely the proper frequency, cesium atoms undergo a transition from one of these states to the other. One **second** (abbreviated s) is defined as the time required for 9,192,631,770 cycles of this microwave radiation (Fig. 3a).

Length

In 1960 an atomic standard for the meter was also established, using the wavelength of the orange-red light emitted by atoms of krypton (^{86}Kr) in a glow discharge tube. Using this length standard, the speed of light in vacuum was measured to be 299,792,458 m/s. In November 1983, the length standard was changed again so that the speed of light in vacuum was *defined* to be precisely

3 The measurements used to determine (a) the duration of a second and (b) the length of a meter. These measurements are useful for setting standards because they give the same results no matter where they are made.

(a) Measuring the second

Microwave radiation with a frequency of exactly 9,192,631,770 cycles per second ...

Outermost electron
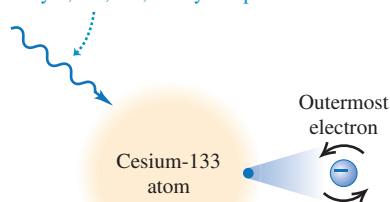
Cesium-133 atom

... causes the outermost electron of a cesium-133 atom to reverse its spin direction.

Cesium-133 atom
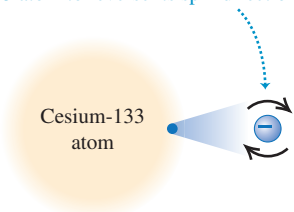
An atomic clock uses this phenomenon to tune microwaves to this exact frequency. It then counts 1 second for each 9,192,631,770 cycles.

(b) Measuring the meter

0:00 s   0:01 s

Light source

Light travels exactly 299,792,458 m in 1 s.

299,792,458 m/s. Hence the new definition of the **meter** (abbreviated m) is the distance that light travels in vacuum in 1/299,792,458 second (Fig. 3b). This provides a much more precise standard of length than the one based on a wavelength of light.

Mass

The standard of mass, the **kilogram** (abbreviated kg), is defined to be the mass of a particular cylinder of platinum–iridium alloy kept at the International Bureau of Weights and Measures at Sèvres, near Paris (Fig. 4). An atomic standard of mass would be more fundamental, but at present we cannot measure masses on an atomic scale with as much accuracy as on a macroscopic scale. The *gram* (which is not a fundamental unit) is 0.001 kilogram.

4 The international standard kilogram is the metal object carefully enclosed within these nested glass containers.

National Institute of Standards and Technology (NIST)

Unit Prefixes

Once we have defined the fundamental units, it is easy to introduce larger and smaller units for the same physical quantities. In the metric system these other units are related to the fundamental units (or, in the case of mass, to the gram) by multiples of 10 or $\frac{1}{10}$. Thus one kilometer (1 km) is 1000 meters, and one centimeter (1 cm) is $\frac{1}{100}$ meter. We usually express multiples of 10 or $\frac{1}{10}$ in exponential notation: $1000 = 10^3$, $\frac{1}{1000} = 10^{-3}$, and so on. With this notation, $1\ \text{km} = 10^3\ \text{m}$ and $1\ \text{cm} = 10^{-2}\ \text{m}$.

The names of the additional units are derived by adding a **prefix** to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, always means a unit larger by a factor of 1000; thus

$$1 \text{ kilometer} = 1 \text{ km} = 10^3 \text{ meters} = 10^3 \text{ m}$$
$$1 \text{ kilogram} = 1 \text{ kg} = 10^3 \text{ grams} = 10^3 \text{ g}$$
$$1 \text{ kilowatt} = 1 \text{ kW} = 10^3 \text{ watts} = 10^3 \text{ W}$$

Table 1 gives some examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time. Figure 5 shows how these prefixes are used to describe both large and small distances.

The British System

Finally, we mention the British system of units. These units are used only in the United States and a few other countries, and in most of these they are being replaced by SI units. British units are now officially defined in terms of SI units, as follows:

Length: 1 inch = 2.54 cm (exactly)

Force: 1 pound = 4.448221615260 newtons (exactly)

Table 1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and living cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^3 m (a 10-minute walk)		

5 Some typical lengths in the universe. **(f)** is a scanning tunneling microscope image of atoms on a crystal surface; **(g)** is an artist's impression.

(a) 10^{26} m
Limit of the observable universe
R. Williams (STScI), the HDF-STeam, and NASA

(b) 10^{11} m
Distance to the sun
SOHO (ESA& NASA)

(c) 10^7 m
Diameter of the earth
Courtesy of NASA/JPL/Caltech

(d) 1 m
Human dimensions
Photodisc Green/Getty Images

(e) 10^{-5} m
Diameter of a red blood cell
PhotoDisc/ GettyImages

(f) 10^{-10} m
Radius of an atom
Purdue University. Veeco Instruments, Inc.

(g) 10^{-14} m
Radius of an atomic nucleus
SPL/PhotoResearchers, Inc.

6 Many everyday items make use of both SI and British units. An example is this speedometer from a U.S.-built automobile, which shows the speed in both kilometers per hour (inner scale) and miles per hour (outer scale).

The newton, abbreviated N, is the SI unit of force. The British unit of time is the second, defined the same way as in SI. In physics, British units are used only in mechanics and thermodynamics; there is no British system of electrical units.

In this text we use SI units for all examples and problems, but we occasionally give approximate equivalents in British units. As you do problems using SI units, you may also wish to convert to the approximate British equivalents if they are more familiar to you (Fig. 6). But you should try to *think* in SI units as much as you can.

4 Unit Consistency and Conversions

We use equations to express relationships among physical quantities, represented by algebraic symbols. Each algebraic symbol always denotes both a number and a unit. For example, d might represent a distance of 10 m, t a time of 5 s, and v a speed of 2 m/s.

An equation must always be **dimensionally consistent.** You can't add apples and automobiles; two terms may be added or equated only if they have the same units. For example, if a body moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt$$

If d is measured in meters, then the product vt must also be expressed in meters. Using the above numbers as an example, we may write

$$10 \text{ m} = \left(2 \frac{\text{m}}{\text{s}} \right) (5 \text{ s})$$

Because the unit 1/s on the right side of the equation cancels the unit s, the product has units of meters, as it must. In calculations, units are treated just like algebraic symbols with respect to multiplication and division.

CAUTION **Always use units in calculations** When a problem requires calculations using numbers with units, *always* write the numbers with the correct units and carry the units through the calculation as in the example above. This provides a very useful check. If at some stage in a calculation you find that an equation or an expression has inconsistent units, you know you have made an error somewhere. In this text we will *always* carry units through all calculations, and we strongly urge you to follow this practice when you solve problems. ▮

Pearson Education

Problem-Solving Strategy 2 — Solving Physics Problems

IDENTIFY *the relevant concepts:* In most cases, it's best to use the fundamental SI units (lengths in meters, masses in kilograms, and times in seconds) in every problem. If you need the answer to be in a different set of units (such as kilometers, grams, or hours), wait until the end of the problem to make the conversion.

SET UP *the problem* and **EXECUTE** *the solution:* Units are multiplied and divided just like ordinary algebraic symbols. This gives us an easy way to convert a quantity from one set of units to another: Express the same physical quantity in two different units and form an equality.

For example, when we say that 1 min = 60 s, we don't mean that the number 1 is equal to the number 60; rather, we mean that 1 min represents the same physical time interval as 60 s. For this reason, the ratio $(1\text{ min})/(60\text{ s})$ equals 1, as does its reciprocal $(60\text{ s})/(1\text{ min})$. We may multiply a quantity by either of these factors (which we call *unit multipliers*) without changing that quantity's physical meaning. For example, to find the number of seconds in 3 min, we write

$$3\text{ min} = (3\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right) = 180\text{ s}$$

EVALUATE *your answer:* If you do your unit conversions correctly, unwanted units will cancel, as in the example above. If, instead, you had multiplied 3 min by $(1\text{ min})/(60\text{ s})$, your result would have been the nonsensical $\frac{1}{20}\text{ min}^2/\text{s}$. To be sure you convert units properly, you must write down the units at *all* stages of the calculation.

Finally, check whether your answer is reasonable. For example, the result 3 min = 180 s is reasonable because the second is a smaller unit than the minute, so there are more seconds than minutes in the same time interval.

Example 1 — Converting speed units

The world land speed record is 763.0 mi/h, set on October 15, 1997, by Andy Green in the jet-engine car *Thrust SSC.* Express this speed in meters per second.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We need to convert the units of a speed from mi/h to m/s. We must therefore find unit multipliers that relate (i) miles to meters and (ii) hours to seconds. We can find the equalities 1 mi = 1.609 km, 1 km = 1000 m, and 1 h = 3600 s. We set up the conversion as follows, which ensures that all the desired cancellations by division take place:

$$763.0\text{ mi/h} = \left(763.0\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{1.609\text{ km}}{1\text{ mi}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$$
$$= 341.0\text{ m/s}$$

EVALUATE: Green's was the first supersonic land speed record (the speed of sound in air is about 340 m/s). This example shows a useful rule of thumb: A speed expressed in m/s is a bit less than half the value expressed in mi/h, and a bit less than one-third the value expressed in km/h. For example, a normal freeway speed is about 30 m/s = 67 mi/h = 108 km/h, and a typical walking speed is about 1.4 m/s = 3.1 mi/h = 5.0 km/h.

Example 2 — Converting volume units

The world's largest cut diamond is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Here we are to convert the units of a volume from cubic inches (in.^3) to both cubic centimeters (cm^3) and cubic meters (m^3). Recall that 1 in. = 2.540 cm, from which we obtain $1\text{ in.}^3 = (2.54\text{ cm})^3$. We then have

$$1.84\text{ in.}^3 = (1.84\text{ in.}^3)\left(\frac{2.54\text{ cm}}{1\text{ in.}}\right)^3$$
$$= (1.84)(2.54)^3\ \frac{\text{in.}^3\ \text{cm}^3}{\text{in.}^3} = 30.2\text{ cm}^3$$

Recall that 1 m = 100 cm, so

$$30.2\text{ cm}^3 = (30.2\text{ cm}^3)\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3$$
$$= (30.2)\left(\frac{1}{100}\right)^3\frac{\text{cm}^3\ \text{m}^3}{\text{cm}^3} = 30.2\times10^{-6}\text{ m}^3$$
$$= 3.02\times10^{-5}\text{ m}^3$$

EVALUATE: Following the pattern of these conversions, you can show that $1\text{ in.}^3 \approx 16\text{ cm}^3$ and that $1\text{ m}^3 \approx 60{,}000\text{ in.}^3$. These approximate unit conversions may be useful for future reference.

15

5 Uncertainty and Significant Figures

7 This spectacular mishap was the result of a very small percent error—traveling a few meters too far at the end of a journey of hundreds of thousands of meters.

Liaison/Getty Images

Table 2 Using Significant Figures

Multiplication or division:
Result may have no more significant figures than the starting number with the fewest significant figures:

$$\frac{0.745 \times 2.2}{3.885} = 0.42$$

$$1.32578 \times 10^7 \times 4.11 \times 10^{-3} = 5.45 \times 10^4$$

Addition or subtraction:
Number of significant figures is determined by the starting number with the largest uncertainty (i.e., fewest digits to the right of the decimal point):

$$27.153 + 138.2 - 11.74 = 153.6$$

8 Determining the value of π from the circumference and diameter of a circle.

135 mm

424 mm

The measured values have only three significant figures, so their calculated ratio (π) also has only three significant figures.

Measurements always have uncertainties. If you measure the thickness of the cover of a hardbound version of a book using an ordinary ruler, your measurement is reliable only to the nearest millimeter, and your result will be 3 mm. It would be *wrong* to state this result as 3.00 mm; given the limitations of the measuring device, you can't tell whether the actual thickness is 3.00 mm, 2.85 mm, or 3.11 mm. But if you use a micrometer caliper, a device that measures distances reliably to the nearest 0.01 mm, the result will be 2.91 mm. The distinction between these two measurements is in their **uncertainty.** The measurement using the micrometer caliper has a smaller uncertainty; it's a more accurate measurement. The uncertainty is also called the **error** because it indicates the maximum difference there is likely to be between the measured value and the true value. The uncertainty or error of a measured value depends on the measurement technique used.

We often indicate the **accuracy** of a measured value—that is, how close it is likely to be to the true value—by writing the number, the symbol $\pm$, and a second number indicating the uncertainty of the measurement. If the diameter of a steel rod is given as 56.47 ± 0.02 mm, this means that the true value is unlikely to be less than 56.45 mm or greater than 56.49 mm. In a commonly used shorthand notation, the number 1.6454(21) means 1.6454 ± 0.0021. The numbers in parentheses show the uncertainty in the final digits of the main number.

We can also express accuracy in terms of the maximum likely **fractional error** or **percent error** (also called *fractional uncertainty* and *percent uncertainty*). A resistor labeled "47 ohms $\pm$ 10%" probably has a true resistance that differs from 47 ohms by no more than 10% of 47 ohms—that is, by about 5 ohms. The resistance is probably between 42 and 52 ohms. For the diameter of the steel rod given above, the fractional error is $(0.02 \text{ mm})/(56.47 \text{ mm})$, or about 0.0004; the percent error is $(0.0004)(100\%)$, or about 0.04%. Even small percent errors can sometimes be very significant (Fig. 7).

In many cases the uncertainty of a number is not stated explicitly. Instead, the uncertainty is indicated by the number of meaningful digits, or **significant figures,** in the measured value. We gave the thickness of the cover of a book as 2.91 mm, which has three significant figures. By this we mean that the first two digits are known to be correct, while the third digit is uncertain. The last digit is in the hundredths place, so the uncertainty is about 0.01 mm. Two values with the *same* number of significant figures may have *different* uncertainties; a distance given as 137 km also has three significant figures, but the uncertainty is about 1 km.

When you use numbers that have uncertainties to compute other numbers, the computed numbers are also uncertain. When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. For example, $3.1416 \times 2.34 \times 0.58 = 4.3$. When we add and subtract numbers, it's the location of the decimal point that matters, not the number of significant figures. For example, $123.62 + 8.9 = 132.5$. Although 123.62 has an uncertainty of about 0.01, 8.9 has an uncertainty of about 0.1. So their sum has an uncertainty of about 0.1 and should be written as 132.5, not 132.52. Table 2 summarizes these rules for significant figures.

As an application of these ideas, suppose you want to verify the value of π, the ratio of the circumference of a circle to its diameter. The true value of this ratio to ten digits is 3.141592654. To test this, you draw a large circle and measure its circumference and diameter to the nearest millimeter, obtaining the values 424 mm and 135 mm (Fig. 8). You punch these into your calculator and obtain the quotient $(424 \text{ mm})/(135 \text{ mm}) = 3.140740741$. This may seem to disagree with the true value of π, but keep in mind that each of your measurements has three significant figures, so your measured value of π can have only three significant figures. It should be stated simply as 3.14. Within the limit of three significant figures, your value does agree with the true value.

In the examples and problems in this text we usually give numerical values with three significant figures, so your answers should usually have no more than three significant figures. (Many numbers in the real world have even less accuracy. An automobile speedometer, for example, usually gives only two significant figures.) Even if you do the arithmetic with a calculator that displays ten digits, it would be wrong to give a ten-digit answer because it misrepresents the accuracy of the results. Always round your final answer to keep only the correct number of significant figures or, in doubtful cases, one more at most. In Example 1 it would have been wrong to state the answer as 341.01861 m/s. Note that when you reduce such an answer to the appropriate number of significant figures, you must *round,* not *truncate.* Your calculator will tell you that the ratio of 525 m to 311 m is 1.688102894; to three significant figures, this is 1.69, not 1.68.

When we calculate with very large or very small numbers, we can show significant figures much more easily by using **scientific notation,** sometimes called **powers-of-10 notation.** The distance from the earth to the moon is about 384,000,000 m, but writing the number in this form doesn't indicate the number of significant figures. Instead, we move the decimal point eight places to the left (corresponding to dividing by 10^8) and multiply by 10^8; that is,

$$384{,}000{,}000 \text{ m} = 3.84 \times 10^8 \text{ m}$$

In this form, it is clear that we have three significant figures. The number 4.00×10^{-7} also has three significant figures, even though two of them are zeros. Note that in scientific notation the usual practice is to express the quantity as a number between 1 and 10 multiplied by the appropriate power of 10.

When an integer or a fraction occurs in a general equation, we treat that number as having no uncertainty at all. For example, in the equation $v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0)$, the coefficient 2 is *exactly* 2. We can consider this coefficient as having an infinite number of significant figures $(2.000000\ldots)$. The same is true of the exponent 2 in $v_x{}^2$ and $v_{0x}{}^2$.

Finally, let's note that **precision** is not the same as *accuracy.* A cheap digital watch that gives the time as 10:35:17 A.M. is very *precise* (the time is given to the second), but if the watch runs several minutes slow, then this value isn't very *accurate.* On the other hand, a grandfather clock might be very accurate (that is, display the correct time), but if the clock has no second hand, it isn't very precise. A high-quality measurement is both precise *and* accurate.

Example 3 **Significant figures in multiplication**

The rest energy E of an object with rest mass m is given by Einstein's famous equation $E = mc^2$, where c is the speed of light in vacuum. Find E for an electron for which (to three significant figures) $m = 9.11 \times 10^{-31}$ kg. The SI unit for E is the joule (J); $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.

SOLUTION

IDENTIFY and SET UP: Our target variable is the energy E. We are given the value of the mass m; from Section 3 the speed of light is $c = 2.99792458 \times 10^8$ m/s.

EXECUTE: Substituting the values of m and c into Einstein's equation, we find

$$
\begin{aligned}
E &= (9.11 \times 10^{-31} \text{ kg})(2.99792458 \times 10^8 \text{ m/s})^2 \\
&= (9.11)(2.99792458)^2(10^{-31})(10^8)^2 \text{ kg} \cdot \text{m}^2/\text{s}^2 \\
&= (81.87659678)(10^{[-31+(2\times 8)]}) \text{ kg} \cdot \text{m}^2/\text{s}^2 \\
&= 8.187659678 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2
\end{aligned}
$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 = 8.19 \times 10^{-14} \text{ J}$$

EVALUATE: While the rest energy contained in an electron may seem ridiculously small, on the atomic scale it is tremendous. Compare our answer to 10^{-19} J, the energy gained or lost by a single atom during a typical chemical reaction. The rest energy of an electron is about 1,000,000 times larger!

Test Your Understanding of Section 5 The density of a material is equal to its mass divided by its volume. What is the density (in kg/m^3) of a rock of mass 1.80 kg and volume 6.0×10^{-4} m^3? (i) 3×10^3 kg/m^3; (ii) 3.0×10^3 kg/m^3; (iii) 3.00×10^3 kg/m^3; (iv) 3.000×10^3 kg/m^3; (v) any of these—all of these answers are mathematically equivalent.

6 Estimates and Orders of Magnitude

PhET: Estimation

We have stressed the importance of knowing the accuracy of numbers that represent physical quantities. But even a very crude estimate of a quantity often gives us useful information. Sometimes we know how to calculate a certain quantity, but we have to guess at the data we need for the calculation. Or the calculation might be too complicated to carry out exactly, so we make some rough approximations. In either case our result is also a guess, but such a guess can be useful even if it is uncertain by a factor of two, ten, or more. Such calculations are often called **order-of-magnitude estimates.** The great Italian-American nuclear physicist Enrico Fermi (1901–1954) called them "back-of-the-envelope calculations."

Exercises 16 through 25 at the end of this chapter are of the estimating, or order-of-magnitude, variety. Most require guesswork for the needed input data. Don't try to look up a lot of data; make the best guesses you can. Even when they are off by a factor of ten, the results can be useful and interesting.

Example 4 **An order-of-magnitude estimate**

You are writing an adventure novel in which the hero escapes across the border with a billion dollars' worth of gold in his suitcase. Could anyone carry that much gold? Would it fit in a suitcase?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Gold sells for around $400 an ounce. (The price has varied between $200 and $1000 over the past decade or so.) An ounce is about 30 grams; that's worth remembering. So ten dollars' worth of gold has a mass of $\frac{1}{40}$ ounce, or around one gram. A billion (10^9) dollars' worth of gold is a hundred million (10^8) grams, or a hundred thousand (10^5) kilograms. This corresponds to a weight in British units of around 200,000 lb, or 100 tons. No human hero could lift that weight!

Roughly what is the *volume* of this gold? The density of gold is much greater than that of water (1 g/cm^3), or 1000 kg/m^3; if its density is 10 times that of water, this much gold will have a volume of 10 m^3, many times the volume of a suitcase.

EVALUATE: Clearly your novel needs rewriting. Try the calculation again with a suitcase full of five-carat (1-gram) diamonds, each worth $100,000. Would this work?

Application Scalar Temperature, Vector Wind

This weather station measures temperature, a scalar quantity that can be positive or negative (say, $+20°C$ or $-5°C$) but has no direction. It also measures wind velocity, which is a vector quantity with both magnitude and direction (for example, 15 km/h from the west).

Scott Bauer - USDAAgricultural Research Service

Test Your Understanding of Section 6 Can you estimate the total number of teeth in all the mouths of everyone (students, staff, and faculty) on your campus? (*Hint:* How many teeth are in your mouth? Count them!)

7 Vectors and Vector Addition

Some physical quantities, such as time, temperature, mass, and density, can be described completely by a single number with a unit. But many other important quantities in physics have a *direction* associated with them and cannot be described by a single number. A simple example is describing the motion of an airplane: We must say not only how fast the plane is moving but also in what direction. The speed of the airplane combined with its direction of motion together constitute a quantity called *velocity*. Another example is *force,* which in physics means a push or pull exerted on a body. Giving a complete description of a force means describing both how hard the force pushes or pulls on the body and the direction of the push or pull.

When a physical quantity is described by a single number, we call it a **scalar quantity.** In contrast, a **vector quantity** has both a **magnitude** (the "how much" or "how big" part) and a direction in space. Calculations that combine scalar quantities use the operations of ordinary arithmetic. For example, 6 kg + 3 kg = 9 kg, or 4 × 2 s = 8 s. However, combining vectors requires a different set of operations.

To understand more about vectors and how they combine, we start with the simplest vector quantity, **displacement.** Displacement is simply a change in the position of an object. Displacement is a vector quantity because we must state not only how far the object moves but also in what direction. Walking 3 km north from your front door doesn't get you to the same place as walking 3 km southeast; these two displacements have the same magnitude but different directions.

We usually represent a vector quantity such as displacement by a single letter, such as $\vec{A}$ in Fig. 9a. In this text we always print vector symbols in **_boldface italic type with an arrow above them._** We do this to remind you that vector quantities have different properties from scalar quantities; the arrow is a reminder that vectors have direction. When you handwrite a symbol for a vector, _always_ write it with an arrow on top. If you don't distinguish between scalar and vector quantities in your notation, you probably won't make the distinction in your thinking either, and hopeless confusion will result.

We always _draw_ a vector as a line with an arrowhead at its tip. The length of the line shows the vector's magnitude, and the direction of the line shows the vector's direction. Displacement is always a straight-line segment directed from the starting point to the ending point, even though the object's actual path may be curved (Fig. 9b). Note that displacement is not related directly to the total _distance_ traveled. If the object were to continue on past P_2 and then return to P_1, the displacement for the entire trip would be _zero_ (Fig. 9c).

If two vectors have the same direction, they are **parallel.** If they have the same magnitude _and_ the same direction, they are _equal,_ no matter where they are located in space. The vector $\vec{A}'$ from point P_3 to point P_4 in Fig. 10 has the same length and direction as the vector $\vec{A}$ from P_1 to P_2. These two displacements are equal, even though they start at different points. We write this as $\vec{A}' = \vec{A}$ in Fig. 10; the boldface equals sign emphasizes that equality of two vector quantities is not the same relationship as equality of two scalar quantities. Two vector quantities are equal only when they have the same magnitude _and_ the same direction.

The vector $\vec{B}$ in Fig. 10, however, is not equal to $\vec{A}$ because its direction is _opposite_ to that of $\vec{A}$. We define the **negative of a vector** as a vector having the same magnitude as the original vector but the _opposite_ direction. The negative of vector quantity $\vec{A}$ is denoted as $-\vec{A}$, and we use a boldface minus sign to emphasize the vector nature of the quantities. If $\vec{A}$ is 87 m south, then $-\vec{A}$ is 87 m north. Thus we can write the relationship between $\vec{A}$ and $\vec{B}$ in Fig. 10 as $\vec{A} = -\vec{B}$ or $\vec{B} = -\vec{A}$. When two vectors $\vec{A}$ and $\vec{B}$ have opposite directions, whether their magnitudes are the same or not, we say that they are **antiparallel.**

We usually represent the _magnitude_ of a vector quantity (in the case of a displacement vector, its length) by the same letter used for the vector, but in _light italic type_ with _no_ arrow on top. An alternative notation is the vector symbol with vertical bars on both sides:

$$(\text{Magnitude of } \vec{A}) = A = |\vec{A}| \qquad (1)$$

The magnitude of a vector quantity is a scalar quantity (a number) and is _always positive._ Note that a vector can never be equal to a scalar because they are different kinds of quantities. The expression "$\vec{A} = 6$ m" is just as wrong as "2 oranges = 3 apples"!

When drawing diagrams with vectors, it's best to use a scale similar to those used for maps. For example, a displacement of 5 km might be represented in a diagram by a vector 1 cm long, and a displacement of 10 km by a vector 2 cm long. In a diagram for velocity vectors, a vector that is 1 cm long might represent

9 Displacement as a vector quantity. A displacement is always a straight-line segment directed from the starting point to the ending point, even if the path is curved.

(a) We represent a displacement by an arrow pointing in the direction of displacement.

Ending position: P_2

Displacement $\vec{A}$

Starting position: P_1

Handwritten notation: $\vec{A}$
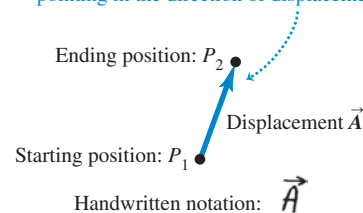
(b) Displacement depends only on the starting and ending positions—not on the path taken.

P_2

$\vec{A}$

Path taken

P_1
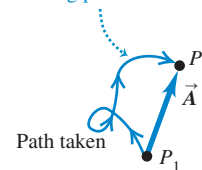
(c) Total displacement for a round trip is 0, regardless of the distance traveled.

P_1
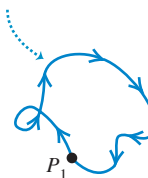
10 The meaning of vectors that have the same magnitude and the same or opposite direction.

P_2 P_4 P_5

$\vec{A}$ $\vec{A}' = \vec{A}$ $\vec{B} = -\vec{A}$

P_1 P_3 P_6
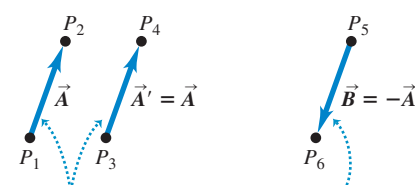
Displacements $\vec{A}$ and $\vec{A}'$ are equal because they have the same length and direction.

Displacement $\vec{B}$ has the same magnitude as $\vec{A}$ but opposite direction; $\vec{B}$ is the negative of $\vec{A}$.

19

11 Three ways to add two vectors. As shown in (b), the order in vector addition doesn't matter; vector addition is commutative.

(a) We can add two vectors by placing them head to tail.
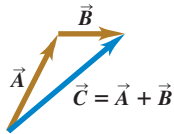
(b) Adding them in reverse order gives the same result.
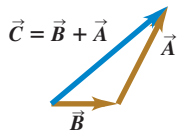
(c) We can also add them by constructing a parallelogram.
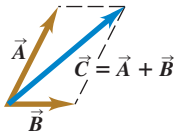
12 (a) Only when two vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their sum equal the sum of their magnitudes: $C = A + B$. (b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their sum equals the *difference* of their magnitudes: $C = |A - B|$.

(a) The sum of two parallel vectors

(b) The sum of two antiparallel vectors

a velocity of magnitude 5 m/s. A velocity of 20 m/s would then be represented by a vector 4 cm long.

Vector Addition and Subtraction

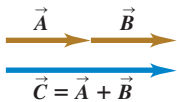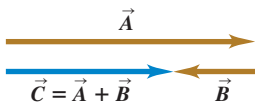Suppose a particle undergoes a displacement $\vec{A}$ followed by a second displacement $\vec{B}$. The final result is the same as if the particle had started at the same initial point and undergone a single displacement $\vec{C}$ (Fig. 11a). We call displacement $\vec{C}$ the **vector sum,** or **resultant,** of displacements $\vec{A}$ and $\vec{B}$. We express this relationship symbolically as

$$\vec{C} = \vec{A} + \vec{B} \qquad (2)$$

The boldface plus sign emphasizes that adding two vector quantities requires a geometrical process and is not the same operation as adding two scalar quantities such as $2 + 3 = 5$. In vector addition we usually place the *tail* of the *second* vector at the *head,* or tip, of the *first* vector (Fig. 11a).

If we make the displacements $\vec{A}$ and $\vec{B}$ in reverse order, with $\vec{B}$ first and $\vec{A}$ second, the result is the same (Fig. 11b). Thus

$$\vec{C} = \vec{B} + \vec{A} \quad \text{and} \quad \vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad (3)$$

This shows that the order of terms in a vector sum doesn't matter. In other words, vector addition obeys the commutative law.

Figure 11c shows another way to represent the vector sum: If vectors $\vec{A}$ and $\vec{B}$ are both drawn with their tails at the same point, vector $\vec{C}$ is the diagonal of a parallelogram constructed with $\vec{A}$ and $\vec{B}$ as two adjacent sides.

CAUTION **Magnitudes in vector addition** It's a common error to conclude that if $\vec{C} = \vec{A} + \vec{B}$, then the magnitude C should equal the magnitude A plus the magnitude B. In general, this conclusion is *wrong;* for the vectors shown in Fig. 11, you can see that $C < A + B$. The magnitude of $\vec{A} + \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ *and* on the angle between $\vec{A}$ and $\vec{B}$. Only in the special case in which $\vec{A}$ and $\vec{B}$ are *parallel* is the magnitude of $\vec{C} = \vec{A} + \vec{B}$ equal to the sum of the magnitudes of $\vec{A}$ and $\vec{B}$ (Fig. 12a). When the vectors are *antiparallel* (Fig. 12b), the magnitude of $\vec{C}$ equals the *difference* of the magnitudes of $\vec{A}$ and $\vec{B}$. Be careful about distinguishing between scalar and vector quantities, and you'll avoid making errors about the magnitude of a vector sum. ▮

When we need to add more than two vectors, we may first find the vector sum of any two, add this vectorially to the third, and so on. Figure 13a shows three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. In Fig. 13b we first add $\vec{A}$ and $\vec{B}$ to give a vector sum $\vec{D}$; we then add vectors $\vec{C}$ and $\vec{D}$ by the same process to obtain the vector sum $\vec{R}$:

$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} = \vec{D} + \vec{C}$$

13 Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.

(a) To find the sum of these three vectors ...
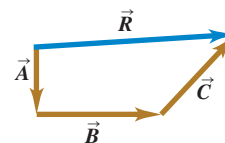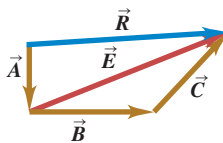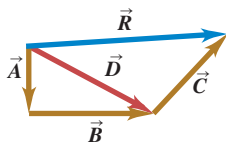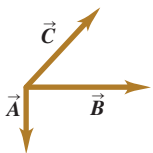
(b) we could add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$, ...

(c) or we could add $\vec{B}$ and $\vec{C}$ to get $\vec{E}$ and then add $\vec{A}$ to $\vec{E}$ to get $\vec{R}$, ...

(d) or we could add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly, ...

(e) or we could add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.

14 To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.

Subtracting $\vec{B}$ from $\vec{A}$ ...   ... is equivalent to adding $-\vec{B}$ to $\vec{A}$. $\vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

With $\vec{A}$ and $-\vec{B}$ head to tail, $\vec{A} - \vec{B}$ is the vector from the tail of $\vec{A}$ to the head of $-\vec{B}$.

With $\vec{A}$ and $\vec{B}$ head to head, $\vec{A} - \vec{B}$ is the vector from the tail of $\vec{A}$ to the tail of $\vec{B}$.
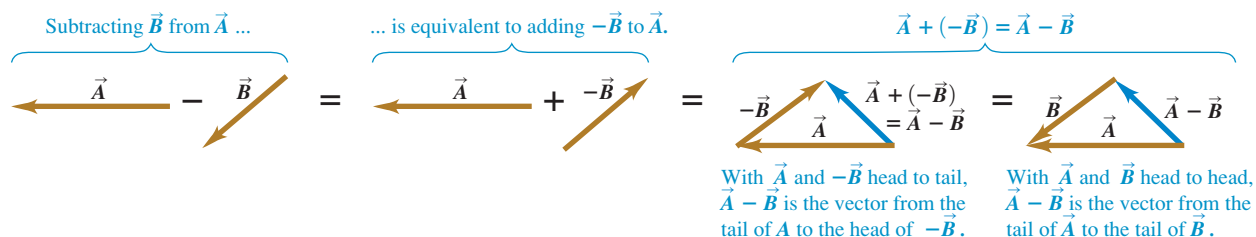
Alternatively, we can first add $\vec{B}$ and $\vec{C}$ to obtain vector $\vec{E}$ (Fig. 13c), and then add $\vec{A}$ and $\vec{E}$ to obtain $\vec{R}$:

$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{E}$$

We don't even need to draw vectors $\vec{D}$ and $\vec{E}$; all we need to do is draw $\vec{A}$, $\vec{B}$, and $\vec{C}$ in succession, with the tail of each at the head of the one preceding it. The sum vector $\vec{R}$ extends from the tail of the first vector to the head of the last vector (Fig. 13d). The order makes no difference; Fig. 13e shows a different order, and we invite you to try others. We see that vector addition obeys the associative law.

We can *subtract* vectors as well as add them. To see how, recall that vector $-\vec{A}$ has the same magnitude as $\vec{A}$ but the opposite direction. We define the difference $\vec{A} - \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ to be the vector sum of $\vec{A}$ and $-\vec{B}$:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \tag{4}$$

Figure 14 shows an example of vector subtraction.

A vector quantity such as a displacement can be multiplied by a scalar quantity (an ordinary number). The displacement $2\vec{A}$ is a displacement (vector quantity) in the same direction as the vector $\vec{A}$ but twice as long; this is the same as adding $\vec{A}$ to itself (Fig. 15a). In general, when a vector $\vec{A}$ is multiplied by a scalar c, the result $c\vec{A}$ has magnitude $|c|A$ (the absolute value of c multiplied by the magnitude of the vector $\vec{A}$). If c is positive, $c\vec{A}$ is in the same direction as $\vec{A}$; if c is negative, $c\vec{A}$ is in the direction opposite to $\vec{A}$. Thus $3\vec{A}$ is parallel to $\vec{A}$, while $-3\vec{A}$ is antiparallel to $\vec{A}$ (Fig. 15b).

A scalar used to multiply a vector may also be a physical quantity. For example, you may be familiar with the relationship $\vec{F} = m\vec{a}$; the net force $\vec{F}$ (a vector quantity) that acts on a body is equal to the product of the body's mass m (a scalar quantity) and its acceleration $\vec{a}$ (a vector quantity). The direction of $\vec{F}$ is the same as that of $\vec{a}$ because m is positive, and the magnitude of $\vec{F}$ is equal to the mass m (which is positive) multiplied by the magnitude of $\vec{a}$. The unit of force is the unit of mass multiplied by the unit of acceleration.

PhET: Vector Addition

15 Multiplying a vector (a) by a positive scalar and (b) by a negative scalar.

(a) Multiplying a vector by a positive scalar changes the magnitude (length) of the vector, but not its direction.

$2\vec{A}$ is twice as long as $\vec{A}$.
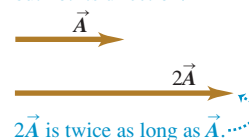
(b) Multiplying a vector by a negative scalar changes its magnitude and reverses its direction.

$-3\vec{A}$ is three times as long as $\vec{A}$ and points in the opposite direction.
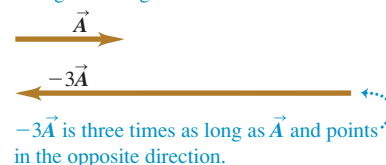
Example 5 **Addition of two vectors at right angles**

A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snowfield. How far and in what direction is she from the starting point?

SOLUTION

IDENTIFY and SET UP: The problem involves combining two displacements at right angles to each other. In this case, vector addition amounts to solving a right triangle, which we can do using the Pythagorean theorem and simple trigonometry. The target variables are the skier's straight-line distance and direction from her

starting point. Figure 16 is a scale diagram of the two displacements and the resultant net displacement. We denote the direction from the starting point by the angle ϕ (the Greek letter phi). The displacement appears to be about 2.4 km. Measurement with a protractor indicates that ϕ is about 63°.
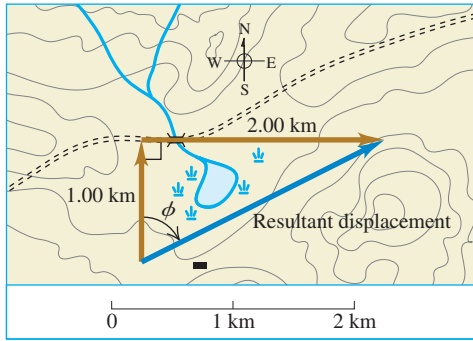
EXECUTE: The distance from the starting point to the ending point is equal to the length of the hypotenuse:

$$\sqrt{(1.00 \text{ km})^2 + (2.00 \text{ km})^2} = 2.24 \text{ km}$$

Continued

16 The vector diagram, drawn to scale, for a ski trip.

2.00 km

1.00 km

ϕ

Resultant displacement

0   1 km   2 km

A little trigonometry (from Appendix B) allows us to find angle ϕ:

$$\tan \phi = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{2.00 \text{ km}}{1.00 \text{ km}}$$

$$\phi = 63.4°$$

We can describe the direction as $63.4°$ east of north or $90° - 63.4° = 26.6°$ north of east.

EVALUATE: Our answers (2.24 km and $\phi = 63.4°$) are close to our predictions. In the more general case in which you have to add two vectors *not* at right angles to each other, you can use the law of cosines in place of the Pythagorean theorem and use the law of sines to find an angle corresponding to ϕ in this example. We'll see more techniques for vector addition in Section 8.

Test Your Understanding of Section 7 Two displacement vectors, $\vec{S}$ and $\vec{T}$, have magnitudes $S = 3$ m and $T = 4$ m. Which of the following could be the magnitude of the difference vector $\vec{S} - \vec{T}$? (There may be more than one correct answer.) (i) 9 m; (ii) 7 m; (iii) 5 m; (iv) 1 m; (v) 0 m; (vi) -1 m.

MP

8 Components of Vectors

In Section 7 we added vectors by using a scale diagram and by using properties of right triangles. Measuring a diagram offers only very limited accuracy, and calculations with right triangles work only when the two vectors are perpendicular. So we need a simple but general method for adding vectors. This is called the method of *components.*

To define what we mean by the components of a vector $\vec{A}$, we begin with a rectangular (Cartesian) coordinate system of axes (Fig. 17a). We then draw the vector with its tail at O, the origin of the coordinate system. We can represent any vector lying in the xy-plane as the sum of a vector parallel to the x-axis and a vector parallel to the y-axis. These two vectors are labeled $\vec{A}_x$ and $\vec{A}_y$ in Fig. 17a; they are called the **component vectors** of vector $\vec{A}$, and their vector sum is equal to $\vec{A}$. In symbols,

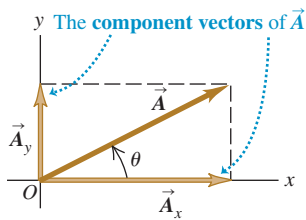$$\vec{A} = \vec{A}_x + \vec{A}_y \tag{5}$$

Since each component vector lies along a coordinate-axis direction, we need only a single number to describe each one. When $\vec{A}_x$ points in the positive x-direction, we define the number A_x to be equal to the magnitude of $\vec{A}_x$. When $\vec{A}_x$ points in the negative x-direction, we define the number A_x to be equal to the negative of that magnitude (the magnitude of a vector quantity is itself never negative). We define the number A_y in the same way. The two numbers A_x and A_y are called the **components** of $\vec{A}$ (Fig. 17b).

CAUTION **Components are not vectors** The components A_x and A_y of a vector $\vec{A}$ are just numbers; they are *not* vectors themselves. This is why we print the symbols for components in light italic type with *no* arrow on top instead of in boldface italic with an arrow, which is reserved for vectors.

We can calculate the components of the vector $\vec{A}$ if we know its magnitude A and its direction. We'll describe the direction of a vector by its angle relative to some reference direction. In Fig. 17b this reference direction is the positive x-axis, and the angle between vector $\vec{A}$ and the positive x-axis

17 Representing a vector $\vec{A}$ in terms of **(a)** component vectors $\vec{A}_x$ and $\vec{A}_y$ and **(b)** components A_x and A_y (which in this case are both positive).

(a)

y The **component vectors** of $\vec{A}$

$\vec{A}$

$\vec{A}_y$

θ

O

$\vec{A}_x$

x

(b)

The **components** of $\vec{A}$

y

$\vec{A}$

$A_y = A\sin\theta$

θ

O

$A_x = A\cos\theta$

x

?